HW # 1

$\boxed{7.}$ $$f_X(x) = \begin{cases} \frac{1}{\theta}, & 0<x<\theta \\ 0, & \text{o.w.} \end{cases}$$

$$F_X(x) = \frac{x}{\theta}$$

$$f_{X_{(1)},X_{(n)}}(x_{(1)}, x_{(n)})$$

$$= \frac{n!}{(n-2)!} f_{X_{(1)}}(x_{(1)}) f_{X_{(n)}}(x_{(n)}) \left[F_{X_{(n)}}(x_{(n)}) - F_{X_{(1)}}(x_{(1)})\right]^{n-2}$$

$$= \frac{n(n-1)}{\theta^n} [x_{(n)} - x_{(1)}]^{n-2}$$

$$u = \frac{x_{(1)}}{x_{(n)}}, \quad v = x_{(n)}$$

$$J = \begin{vmatrix} \frac{\partial x_{(1)}}{\partial u} & \frac{\partial x_{(n)}}{\partial u} \\ \frac{\partial x_{(1)}}{\partial v} & \frac{\partial x_{(n)}}{\partial v} \end{vmatrix} = \begin{vmatrix} v & 0 \\ u & 1 \end{vmatrix} = |v|$$

$$f_{u,v}(u,v) = \frac{n(n-1)}{\theta^n} (v - uv)^{n-2} |J|$$

$$= \frac{n(n-1)}{\theta^n} (1-u)^{n-2} \cdot v^{n-1}$$

It contains the functions of u and v separately, thus we can say that u and v are independent.