1.6 Since we know $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$$E(g(s^2)) = cE(\sqrt{s^2}) = c\cdot\sqrt{\frac{\sigma^2}{n-1}}\,E\left(\sqrt{\frac{S^2(n-1)}{\sigma^2}}\right)$$

where $\sqrt{\frac{(n-1)S^2}{\sigma^2}}$ is a function of $\frac{(n-1)s^2}{\sigma^2}$ thus, let this be z

$$E(g(s^2)) = c\cdot\sqrt{\frac{\sigma^2}{n-1}}\cdot\int_0^\infty \sqrt{z}\,\frac{1}{\Gamma(\frac{n-1}{2})2^{\frac{n-1}{2}}}\cdot z^{\frac{n-1}{2}-1}e^{-\frac{z}{2}}dz$$

$$= c\cdot\sqrt{\frac{\sigma^2}{n-1}}\cdot\frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})}\cdot 2^{\frac{1}{2}}\cdot\underline{\int_0^\infty\frac{1}{\Gamma(\frac{n}{2})2^{\frac{n}{2}}}\cdot z^{\frac{n}{2}-1}e^{-\frac{z}{2}}dz}$$

which formed another Chisq and integrate to 1: Thus,

$$E(g(s^2)) = c\cdot\frac{\sigma}{\sqrt{n-1}}\cdot\frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})}\cdot 2^{\frac{1}{2}}$$

Inorder to make this equal to σ, c has to be:

$$c = \frac{\sqrt{n-1}}{\sqrt{2}}\cdot\frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})}$$

2.3 $$f_{X_i,\theta}(x) = e^{i\theta-x}\cdot I_{[i\theta,+\infty)}(x)$$

The joint density:

$$\prod_{i=1}^n\left\{e^{i\theta}\cdot e^{-X_i}\cdot I_{[i\theta,+\infty)}(X_i)\right\}$$

$$= e^{i\theta\cdot n}\cdot e^{-\sum_{i=1}^n X_i}\prod_{i=1}^n I_{[\theta,+\infty)}\left(\frac{X_i}{i}\right), \quad i>0$$

Thus, all $\frac{X_i}{i}$ has to be $\geq\theta \Rightarrow \min\left(\frac{X_i}{i}\right)\geq\theta$

Then we can see that $h(x) = e^{-\sum_{i=1}^n X_i}$

$$g(T(\underline{X})|\theta) = e^{i\theta n}\cdot\prod_{i=1}^n I_{[\theta,+\infty)}\left(\frac{X_i}{i}\right)$$

$$= e^{i\theta n}\cdot I_{\min_i\{\frac{X_i}{i}\geq\theta\}}$$

Therefore $T(\underline{X}) = \min_i\{\frac{X_i}{i}\}$ is a sufficient statistic for θ