(6) $X_1, \dots X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}, \quad V = \sqrt{\frac{(n-1)S^2}{\sigma^2}} \sim \chi_{n-1}$$

$$E[V] = \sqrt{2} \cdot \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})}$$

$$E[V] = \frac{\sqrt{n-1}\ E[S]}{\sigma} = \sqrt{2}\frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})}$$

$$\Rightarrow \frac{\sqrt{n-1}}{\sqrt{2}} \cdot \frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})} E[S] = \sigma$$

$$\Rightarrow E\left[\sqrt{\frac{n-1}{2}} \cdot \frac{\Gamma(\frac{n-1}{2})}{\Gamma(n/2)} S\right] = \sigma$$

$$\Rightarrow E[g(S^2)] = E\left[\sqrt{\frac{n-1}{2}} \frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})} S\right]$$

$$\Rightarrow g(S^2) = \sqrt{\frac{n-1}{2}} \frac{\Gamma((n-1)/2)}{\Gamma(n/2)} \sqrt{S^2}$$

$$\therefore C = \sqrt{\frac{n-1}{2}} \frac{\Gamma((n-1)/2)}{\Gamma(n/2)}$$

$$※\ \Gamma(\tfrac{n}{2}) = \frac{(n-2)!!\sqrt{\pi}}{2^{(n-1)/2}}, \quad \Gamma(\tfrac{n-1}{2}) = \frac{(n-3)!!\sqrt{\pi}}{2^{(n-2)/2}}$$

$$\frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})} = \frac{\dfrac{(n-3)!!\sqrt{\pi}}{2^{(n-1)/2} \cdot 2^{-1/2}}}{\dfrac{(n-2)!!\sqrt{\pi}}{2^{(n-1)/2}}} = \sqrt{2} \cdot \frac{(n-3)!!}{(n-2)!!}$$

$$C = \frac{\sqrt{n-1}}{\sqrt{2}} \cdot \frac{\Gamma(\frac{n-1}{2})}{\Gamma(n/2)} = \frac{\sqrt{n-1}}{\cancel{\sqrt{2}}} \cdot \cancel{\sqrt{2}} \frac{(n-3)!!}{(n-2)!!} = \frac{\sqrt{n-1}(n-3)!!}{(n-2)!!}$$