AMS 205B Homework 1 Question 1

Let $X_1, \cdots, X_n$ be iid random variables with continuous cdf F_X, and suppose $E(X_i) = \mu$.

Define the random variables $Y_1, \cdots, Y_n$ by

$Y_i = 1$ if $X_i > \mu$; $Y_i = 0$ o.w..

Find the distribution of $\sum_{i=1}^{n} Y_i$.

$$Pr(X_i > \mu) = 1 - Pr(X_i \leq \mu)$$

$$= 1 - F_X(\mu)$$

$$Pr(X_i \leq \mu) = F_X(\mu).$$

Y_i follows Bernoulli distribution with $p = 1 - F_X(\mu)$

$\Rightarrow \sum_{i=1}^{n} Y_i$ follows Binomial distribution

$Bin(n, p)$ where $p = 1 - F_X(\mu)$

$$Pr\left(\sum_{i=1}^{n} Y_i \leq y\right) = \sum_{i=0}^{y} \binom{n}{i} [1 - F_X(\mu)]^i [F_X(\mu)]^{n-i}.$$