

# Assignment 3

January 31, 2017

1. Let  $X_1, \dots, X_n \stackrel{iid}{\sim} U(\theta, \theta + 1)$ . Show that the minimal sufficient statistic  $(X_{(1)}, X_{(n)})$  observed in the class is not complete.
2. Let  $X_1, \dots, X_n \stackrel{iid}{\sim} N(\theta, a\theta^2)$  where  $a$  is a known constant and  $\theta > 0$ . Show that the statistic  $(\bar{X}, S^2)$  is a sufficient statistic for  $\theta$ , but the family of distributions is not complete.
3. Let  $X$  takes values 0, 1, 2 with probabilities  $p, 3p, 4p$ . Determine if the family of distributions of  $X$  is complete.
4. Let  $X_1, \dots, X_n$  be a random sample from the pdf  $f_\mu(x) = e^{-(x-\mu)}$ ,  $-\infty < \mu < x < \infty$ . Show that  $X_{(1)}$  and  $S^2$  are independent.
5. Let  $X_1, \dots, X_n \stackrel{iid}{\sim} Ber(p)$ , and define the function  $h(p) = P_p(\sum_{i=1}^n X_i > X_{n+1})$ , the probability that the sum of first  $n$  observations exceeds  $(n + 1)$  th observation.

(a) Show that

$$T(X_1, \dots, X_{n+1}) = \begin{cases} 1 & \text{if } \sum_{i=1}^n X_i > X_{n+1} \\ 0 & \text{o.w.} \end{cases}$$

is an unbiased estimator of  $h(p)$ .

(b) Find the best unbiased estimator for  $h(p)$ .

6.  $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Gamma}(\alpha, \beta)$ , with  $\alpha$  known. Find the best unbiased estimator of  $1/\beta$ .
7. Suppose that  $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Ber}(p)$ .
- (a) Show that variance of  $\bar{X}$  attains Cramer-Rao lower bound.
- (b) Find the best unbiased estimator of  $p^8$  when  $\sum_{i=1}^n X_i > 8$ .
8.  $X_1, \dots, X_n$  be a random sample from a population with p.d.f.  $f_\theta(x) = \frac{1}{2\theta}$  for  $-\theta < x < \theta$ ,  $\theta > 0$ . Find the best unbiased estimator of  $\theta$ .